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Casimir interaction between normal or superfluid grains in the Fermi sea

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Abstract
We report on a new force that acts on cavities (literally empty regions of space) when they are immersed in a background of non-interacting fermionic matter fields. The interaction follows from the obstructions to the (quantum mechanical) motion of the fermions caused by the presence of bubbles or other (heavy) particles in the Fermi sea, such as, for example, nuclei in the neutron sea in the inner crust of a neutron star or superfluid grains in a normal Fermi liquid. The effect resembles the traditional Casimir interaction between metallic mirrors in the vacuum. However, the fluctuating electromagnetic fields are replaced by fermionic matter fields. We show that the fermionic Casimir problem for a system of spherical cavities can be solved exactly, since the calculation can be mapped onto a quantum mechanical billiard problem of a point-particle scattered off a finite number of non-overlapping spheres or discs. Finally, we generalize the map method to other Casimir systems, especially to the case of a fluctuating massless scalar field between two spheres or a sphere and a plate under Dirichlet boundary conditions.

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1. Introduction

1.1. The original Casimir effect

In 1948, the Dutch physicist H B G Casimir predicted the remarkable effect [1] that two parallel, very closely spaced, uncharged metallic plates attract each other in vacuum. The origin of this force can be traced back to the altered (mode sums over the) zero-point fluctuations of the electromagnetic field that are induced by the presence of the two plates which are added relative to the free case, or rather, which are brought from a large to a small relative separation. The distinctive property of the Casimir effect is that its strength and, perhaps, its sign are geometry dependent (for a review, see [2]). Our aim is to generalize two features of the Casimir effect, the mode sum structure and the geometry dependence, to the *fermionic Casimir effect*, where the fluctuating photons are replaced by (non-relativistic) Fermi matter fields.

1.2. Utilizing the geometry dependence of the Casimir energy

Let us invert the logic and define the Casimir energy as the energy resulting from the *geometry-dependent* part of the density of states (d.o.s.)—a concept that is closely related to the shell correction energy in nuclear physics:

$$\rho(E) \equiv \sum_{E_k} \delta(E - E_k) = \rho_0(E) + \rho_{\text{bulk}}(E) + \delta\rho_{\text{C}}(E, \text{geom.-dep.}). \qquad (1)$$

Here $\{E_k\}$ are the eigenenergies of the modes, ρ_0 is the d.o.s. of the homogeneous background and ρ_{bulk} is the bulk d.o.s. that sums up the excluded volume effects, surface contributions and Friedel oscillations caused by each of the obstacles separately. The remaining part $\delta\rho_{\text{C}}$ is of central interest to us. It is the only term which 'knows' about the relative geometry dependence of the obstacles. Now the Casimir energy can be extracted from the *geometry-dependent* part of the density of states as a simple integral

$$\mathcal{E}_{\text{C}} \equiv \int \mathrm{d}E\, E\delta\rho_{\text{C}}(E, \text{geom.-dep.}) = -\int \mathrm{d}E\, \mathcal{N}_{\text{C}}(E, \text{geom.-dep.}), \qquad (2)$$

where $\mathcal{N}_{\text{C}}(E)$ is the geometry-dependent part of the integrated density of states or number of states, $\mathcal{N}(E) \equiv \sum_{E_k} \Theta(E - E_k) = \int_0^E \mathrm{d}E' \rho(E')$.

1.3. Generalization of the Casimir energy concept to matter fields

Let us assume that space is not 'filled' with fluctuating electromagnetic modes, but with a gas of *non-interacting* (non-relativistic) fermions. Under this scenario, we have the following similarities with the ordinary Casimir effect: in both cases, there exist mode sums, $\sum_k \hbar\omega_k$, with *constant* degeneracy factors. The constant weight of the fluctuating electromagnetic modes can be traced back to the two helicity states of the photon. The constancy in the degeneracy of the fermionic matter modes, on the other hand, follows from Pauli's exclusion principle, where the pertinent weight can be formulated in terms of spin and isospin factors. However, the Casimir mode summation over matter fields differs from the case of fluctuating fields by the presence of a new independent scale in addition to the geometric scales (e.g. the separation L and area A of the plates), namely by the presence of the Fermi energy (i.e. the chemical potential μ at zero temperature).

In the following, we will consider the case of fermionic matter fields located in the space between voids or cavities, such that the matter fields will build up a quantum pressure on the voids. Even if we assume that the matter fields are non-interacting and the voids are non-overlapping, an *effective* interaction between these empty regions of space will still arise in the background of the non-interacting fermionic matter fields. Namely the cavities—depending on their geometric arrangement—can shield the free movement of the matter modes such that a net change in the mode sum over the Fermi states in the Fermi sea results. Note that the modes are on their mass-shell and delocalized.

Applications of this scenario exist e.g. for the inner crust of a neutron star. With increasing distance from the star's surface (inside the star), the nuclei start to lose neutrons due to the growing pressure and density. Bulgac and Magierski argued that the shell correction energies (in other words, the fermionic Casimir energies) of the resulting low-density neutron matter in the presence of bubbles (i.e. nuclei) are of the same order of magnitude [3] as the differences between the usually assumed liquid-drop-model sequence of phases [4]. Thus, disordered lattices of bubbles are in competition with the standard sequence of (i) nuclear drops (nuclei), (ii) nuclear rods, (iii) nuclear plates (all three surrounded by a low-density background of neutrons), (iv) tubes and (v) bubbles (both surrounded by the high-density background of nuclear matter), until finally a uniform nuclear matter phase is reached.

An analogous investigation of bubbles inside a Fermi gas background is also of relevance for the inner *core* of neutron stars. Under the assumption that quark matter does exist and quark droplets can form there, a similar pattern is predicted, with quark droplets (bags) inside hadronic matter taking over the role of the embedded nuclei.

Also in the laboratory, the study of the interaction of cavities inside a uniform fermionic background could be of importance [5]. Examples are C_{60} buckyballs immersed in liquid mercury. The liquid metal itself serves only as a non-rigid neutral background which provides the Fermi gas environment via its conductance electrons, in which the buckyballs 'drill' the voids. Another example would be buckyballs in liquid ^{3}He as Fermi gas. Finally, in the future, boson-condensate droplets immersed in dilute atomic Fermi condensates could serve as systems with which the effective interactions of cavities inside a Fermi gas could be studied in the lab.

2. Casimir calculation mapped onto a scattering problem

Note that the Casimir calculation for fermionic (non-relativistic) matter fields simplifies enormously, because the presence of a second scale, the chemical potential $\mu = \hbar^2 k_F^2 / 2m$ (or the Fermi momentum k_F), provides for a natural UV-cutoff, $\Lambda_{UV} \equiv \mu$ (here m is the mass of the fermion). Thus, the Casimir energy for fermions between two impenetrable (parallel) planes at a distance L is simply given by the chemical potential times a finite function of the dimensionless argument $k_F L$, $\mathcal{E}_C = \mu F(k_F L)$.

For more complicated geometries, the computations become more and more involved as it is the case for the ordinary electromagnetic Casimir effect. However, Casimir calculations of a finite number of immersed non-overlapping *spherical* voids or rods, i.e. spheres and cylinders in three dimensions or discs in two dimensions, are still doable, although the corresponding quantum mechanical problems are not separable any longer. In fact, these calculations simplify because of Krein's trace formula [6] (see [7] for the special case of spherical symmetry)

$$\delta\bar{\rho}(E) = \bar{\rho}(E) - \bar{\rho}_0(E) = \frac{1}{2\pi \mathrm{i}} \frac{\mathrm{d}}{\mathrm{d}E} \operatorname{tr} \ln S_n(E), \tag{3}$$

which links the variation in the level density $\delta\bar{\rho}(E)$ (the difference of the total density of states and the background one) to the energy variation of the total phase shift $\frac{1}{2\mathrm{i}} \ln \det S_n(E)$ of the n-sphere/disc scattering matrix $S_n(E)$. Note that the level densities on the left-hand side are averaged over an energy interval larger than the mean-level spacing in the volume V of the entire system in order to match the *continuous* expression on the right-hand side. In this way, the Casimir calculation is mapped to the quantum mechanical analogue of a classical 'billiard' problem: the hyperbolic or even chaotic scattering of a non-relativistic (or relativistic or even massless) point particle off an assembly of n non-overlapping spheres (or discs) [8–13]. (For simplicity, we consider in the following, with the exception of section 5, only the non-relativistic case of a particle with kinetic energy $E = \hbar^2 k^2 / 2m$ and wave number k.)

Moreover, the geometry-dependent Casimir fluctuations can be extracted from the *multiple*-scattering part of the S-matrix. As shown in [11–13], the determinant of the n-sphere/disc S-matrix separates into a product of the determinants of the one-sphere/disc S-matrices $S_1(E, a_i)$, where a_i is the radius of the single scatterer i, and the ratio of the determinant of the inverse multi-scattering matrix $M(k)$ and its complex conjugate

$$\det S_n(E) = \left\{ \prod_{i=1}^{n} \det S_1(E, a_i) \right\} \frac{\det(M(k^*)^\dagger)}{\det M(k)}. \tag{4}$$

When inserted into Krein's formula, the product over the single-scatterer determinants generates just the bulk (or Weyl term) contribution to the density of states

$$\bar\rho_{\rm bulk}(E, \{a_i\}) \equiv \sum_{i=1}^{n} \bar\rho_{\rm Weyl}(E, a_i) = \frac{1}{2\pi\,{\rm i}} \frac{{\rm d}}{{\rm d}E} \sum_{i=1}^{n} \ln\det S_1(E, a_i), \tag{5}$$

which takes care of the excluded volume terms and the surface terms (including Friedel oscillations). The geometry-dependent part of the d.o.s. $\delta\bar\rho_{\rm C} \equiv \delta\bar\rho_{\rm C}(E, \{a_i\}, \{\vec r_{ij}\})$ is therefore given by a modified Krein equation [5] which is formulated in terms of the inverse multi-scattering matrix $M \equiv M[k(E), \{a_i\}, \{\vec r_{ij}\}]$ instead of the full S-matrix

$$\delta\bar\rho_{\rm C} = \bar\rho(E) - \bar\rho_0(E) - \sum_{i=1}^{n} \bar\rho_{\rm Weyl}(E, a_i) = -\frac{1}{\pi}\,{\rm Im} \frac{{\rm d}}{{\rm d}E} \ln\det M, \tag{6}$$

where $\vec r_{ij}$ are the relative separation vectors between the centres of the spheres (or discs). The pertinent Casimir energy can then be read off from the finite integral

$$\mathcal{E}_{\rm C} = \int_0^\mu {\rm d}E(E-\mu)\delta\bar\rho_C = -\int_0^\mu {\rm d}E\, \overline{\mathcal{N}}_{\rm C}. \tag{7}$$

3. The calculation

Equation (6) does allow us to simplify the problem, since there exists a close-form expression for the inverse multi-scattering matrix for n spheres (of radii a_j and mutual separation $r_{jj'}$, labelled by the indices $j, j' = 1, 2, \ldots, n$) in terms of spherical Bessel and Hankel functions, spherical harmonics and $3j$-symbols, where l, l' and m, m' are total angular momentum and pertinent magnetic quantum numbers, respectively [13]:

$$\begin{aligned} M^{jj'}_{lm,l'm'} &= \delta^{jj'}\delta_{ll'}\delta_{mm'} + (1-\delta^{jj'})\,{\rm i}^{2m+l'-l}\sqrt{4\pi(2l+1)(2l'+1)} \\ &\quad\times \left(\frac{a_j}{a_{j'}}\right)^2 \frac{j_l(ka_j)}{h^{(1)}_{l'}(ka_{j'})} \sum_{l''=0}^{\infty} \sum_{m''=-l'}^{l'} \sqrt{2l''+1}\,{\rm i}^{l''} \begin{pmatrix} l'' & l' & l \\ 0 & 0 & 0 \end{pmatrix} \\ &\quad\times \begin{pmatrix} l'' & l' & l \\ m-m'' & m'' & -m \end{pmatrix} D^{l'}_{m',m''}(j,j') h^{(1)}_{l''}(kr_{jj'}) Y^{m-m''}_{l''}\big(\hat r^{(j)}_{jj'}\big). \end{aligned} \tag{8}$$

The unit vectors $\hat r^{(j)}_{jj'}$ point from the origin of sphere j (as measured in its local coordinate system) to the origin of sphere j'. The local coordinate system of sphere j' is mapped to the one of sphere j with the help of the rotation matrix $D^{l'}_{m',m''}(j, j')$.

For small scatterers, the expression of the multi-scattering matrix even simplifies

$$M^{jj'}(E) \approx \delta^{jj'} - (1-\delta^{jj'}) f^s_j(E) \frac{\exp({\rm i}kr_{jj'})}{r_{jj'}} + \mathcal{O}(p\text{-wave}), \tag{9}$$

since only s-wave scattering is important, i.e. spherical waves modulated by s-wave amplitudes $f_j^s(E)$ propagate between the spheres. The integrated d.o.s. in the case of two small spherical cavities of common radius a and centre-to-centre separation r is [5]

$$\mathcal{N}_{\mathrm{C}}^{\mathrm{oo}}(E) = -\frac{1}{\pi}\,\mathrm{Im}\ln\det M^{\mathrm{oo}}(E) \approx \nu_{\mathrm{deg}}\frac{a^2}{\pi r^2}\sin[2(r-a)k] + \mathcal{O}((ka)^3) \quad (10)$$

where ν_{deg} is the spin/isospin-degeneracy factor. This expression should be compared with the semiclassical approximation that sums up all partial waves

$$\mathcal{N}_{\mathrm{C,sc}}^{\mathrm{oo}}(E) = \nu_{\mathrm{deg}}\frac{a^2}{4\pi r(r-2a)}\sin[2(r-2a)k]. \quad (11)$$

This is the leading contribution to Gutzwiller's trace formula [14], namely of the non-repeated two-bounce periodic orbit between the spheres, with the action $S_{po}(k)/\hbar = 2(r-2a)k$, where $2(r-2a)$ is the length of the geometric path. Note that the semiclassical result is suppressed by a factor of $1/4$ relative to the small-scatterer one.

As shown in [5], the semiclassical result for the case $k > 1/a$ is a very good approximation of the full quantum mechanical result calculated from the exact expression (8) of the two-sphere scattering matrix when plugged into the modified Krein formula (6). Therefore, the Casimir energy for the two spherical cavities inside a non-relativistic non-interacting fermion background can be approximated in terms of a spherical Bessel function j_1 as

$$\mathcal{E}_{\mathrm{C}}^{\mathrm{oo}} = -\int_0^{\mu(k_{\mathrm{F}})} \mathrm{d}E\,\mathcal{N}_{\mathrm{C}}^{oo}(E) \approx -\nu_{\mathrm{deg}}\mu\frac{a^2}{2\pi r(r-2a)}j_1[2(r-2a)k_{\mathrm{F}}], \quad (12)$$

valid for $k_{\mathrm{F}}a > 1$. This expression is long ranged, i.e. $1/L^3$ with $L = r-2a$ (since the on-shell matter modes are delocalized). The Casimir energy of the sphere–plate system

$$\mathcal{E}_{\mathrm{C}}^{\mathrm{o|}} \approx -\nu_{\mathrm{deg}}\mu\frac{a}{2\pi(r-a)}j_1[2(r-a)k_{\mathrm{F}}], \quad (13)$$

scales even as $1/L^2$ with $L = r-a$. Note that in both cases, the two-sphere system or the sphere–plate system, the Casimir energy does not have a fixed sign in contrast to the standard Casimir effect with fluctuating electromagnetic or scalar fields between these obstacles. Instead the sign of the Casimir energy oscillates as a function of the action of the two-bounce orbit. Therefore, by increasing the distance between the cavities, the Casimir energy, which starts out to be attractive, can be made repulsive, and by further increasing this distance, it can become attractive again, with a decreased strength of course. The reason for this new type of behaviour of a Casimir energy is the presence of a new scale in addition to the length scales, namely the chemical potential μ. In fact, the strength of this fermionic Casimir energy scales with the strength of the chemical potential and therefore with the UV-cutoff of the theory. Also this behaviour distinguishes the fermionic Casimir effect from the standard Casimir effect: the latter is governed by the infrared behaviour of the corresponding density of states.

4. Fermionic Casimir effect between superfluid grains

A generalization of the fermionic Casimir effect to the case of superfluid cavities immersed in normal fermionic matter is reported in [15]. The pairing interaction due to the superfluid obstacles leads to enhanced Casimir contributions, because of the dominance of the particle–hole terms over the particle–particle and hole–hole contributions. Semiclassically, this can be explained by the focusing (and only for large separations defocusing) nature of the Andreev reflections [16] (see figure 1) in comparison to the defocusing specular reflections at normal

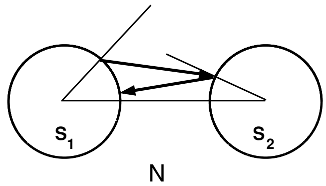

Figure 1. Two superfluid grains (S_1 and S_2) immersed in a normal Fermi liquid N. Indicated is the Andreev reflection between an incoming particle (hole) and an outgoing hole (particle). Compare with figure 1 of [15].

circular boundaries. Since the particle–hole d.o.s has the opposite sign to the particle–particle one and since the d.o.s has to be integrated over the quasi-particle energy as measured from the Fermi surface, the resulting Casimir energy for superfluid obstacles is strongly repulsive [15].

5. Application of the map method to the scalar Dirichlet problem

Note that the map method onto a scattering problem [5] can be generalized from the Casimir effect in the Fermi sea to other systems. It holds, in contrast to the *phase shift method* presented in [17], even for quantum mechanically non-separable problems. It is especially applicable for the case of a fluctuating massless scalar field between two spheres or a sphere and a plate with Dirichlet boundary conditions [18–20]. Whereas the *s*-wave approximation is not important for the fermionic Casimir energy which is governed by the UV part of the d.o.s. (i.e. by the contribution at the Fermi momentum $k_{\rm F}$, assuming $k_{\rm F} > 1/a$), it is essential for the fluctuating-scalar Casimir effect at *very large separations* ($L \gg a$), as this Casimir-energy type is governed by the infrared behaviour of the d.o.s. at $k \sim 1/L$ or less. Remember the relative factor of 4 from [5] between the *s*-wave and semiclassical result of the two-cavity d.o.s.; see (10) and (11). These equations also apply for the *s*-wave and semiclassical d.o.s. of the fluctuating-scalar Casimir effect, if the relativistic dispersion $E = \hbar ck$, the spin/isospin degeneracy $\nu_{\rm deg} \equiv 1$ and the suppression of the repeats of the semiclassical two-bounce orbit at large separations are taken into account [21]. Therefore, the exact result of the fluctuating-scalar Casimir energy for two *very far separated* Dirichlet spheres is enhanced by a factor of 4 relative to the semiclassical result and, moreover, by a factor of $4 \times (90/\pi^4)$ [21] relative to the *leading* term of the proximity-force approximation (PFA) [22]. The leading term of the PFA, in other words the PFA at *vanishing distances*, where the repeats of the two-bounce orbit cannot be neglected and add up to a factor of $\pi^4/90$, was confirmed by the semiclassical calculations of [18].

Correspondingly, as there is only one sphere, the exact result of the Casimir energy of the sphere–plate configuration for $L \gg a$ is $2 \times 90/\pi^4$ times larger than the leading term $\mathcal{E}_{\rm PFA}^{\rm lead} = -\hbar c\pi^3 a/(1440L^2)$ of the PFA and *twice* larger than the full semiclassical result with all repeats. The latter (see (D) of figure 2) interpolates between the PFA at vanishing separation ((E) and (F) of figure 2 are two versions of the PFA [19, 20]) and the value $90/\pi^4$ at large separations. Figure 2 also confirms that the exact calculation (A) and its *s*-wave approximation (B) converge to the asymptotic value $2 \times 90/\pi^4$ (see C).

The exact data (A) are compatible with the numerical data of the worldline approach [19] between $L = a/8$ and $L = a$ if the statistical error bars of the worldline data are taken into account. Moreover, the new (systematically and statistically improved) worldline data

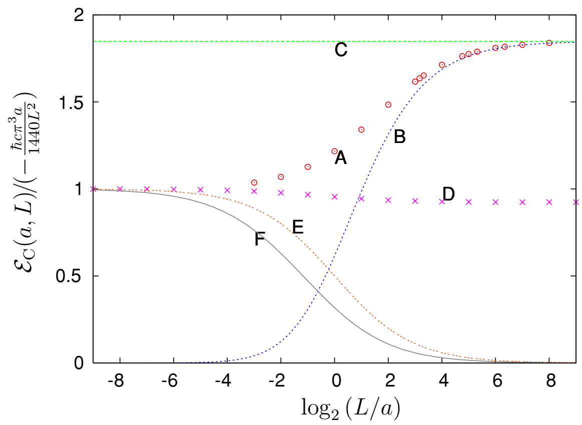

Figure 2. Predictions for the scalar Casimir energy $\mathcal{E}_C(a, L)$ of the Dirichlet sphere–plate configuration in units of the leading term $-\hbar c\pi^3 a/1440L^2$ of the proximity-force approximation as a function of the surface-to-surface distance L, divided by the radius a of the sphere. The points and curves are explained in the text.

presented by H Gies at this QFEXT'05 workshop [23], which extend up to $L = 16a$, do now nicely agree with our exact data, see figure 2 of [21].

In the case of the electromagnetic Casimir effect, the s-wave dominance at large separation has to be replaced by a p-wave dominance, since the charge neutrality of the sphere forbids a monopole term, whereas the standard Casimir–Polder energy is dominated by induced-dipole contributions.

6. Summary

There exists an effective interaction between voids inside a Fermi gas background, even if the fermions are non-interacting. The cavities do not need to be spherical if the curvature radii are larger than the Fermi wavelength. The effects of finite surface thickness can be incorporated into the Weyl contributions and do not affect the Casimir part if the objects do not overlap. Because of the oscillating and long-range nature of the new Casimir interaction (relative to the van der Waals one), disordered lattices are expected as emerging structures. The map method can be generalized to other Casimir systems, especially to the scalar Casimir effect for Dirichlet sphere(s)–plate systems.

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